INVESTIGATION 1-B

Atwood's Machine

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

George Atwood designed his machine to demonstrate the laws of motion. In this investigation, you will demonstrate those laws and determine the value of *g*.

Problem

How can you determine the value of *g*, the acceleration due to gravity, by using an Atwood machine?

Prediction

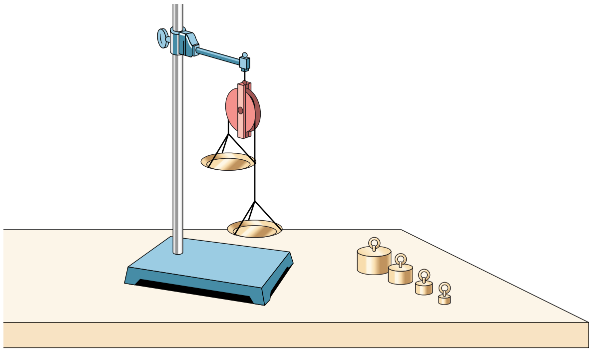
- Predict how changes in the *difference* between the two masses will affect the acceleration of the Atwood machine if the sum of the masses is held constant.
- When the difference between the two masses in an Atwood machine is held constant, predict how increasing the total mass (sum of the two masses) will affect their acceleration.

Equipment

- retort stand
- clamps
- masses: 100 g (2), 20 g (1), 10 g (10), or similar identical masses, such as 1 inch plate washers
- 2 plastic cups to hold masses
- light string

Traditional instrumentation
lab pulley
lab timer
metre stick

Probeware
Smart Pulley® or photogates or ultrasonic range finder
motion analysis software
computer



Procedure

Constant Mass Difference

1. Set up a data table to record m_1, m_2, total mass, Δd and Δt (if you use traditional equipment), and a.
2. Set up an Atwood machine at the edge of a table, so that $m_1 = 120$ g and $m_2 = 100$ g.
3. Lift the heavier mass as close as possible to the pulley. Release the mass and make the measurements necessary for finding its downward acceleration. Catch the mass before it hits the floor.
   - Using traditional equipment, find displacement (Δd) and the time interval (Δt) while the mass descends smoothly.
   - Using probeware, measure velocity (v) and graph velocity versus time. Find acceleration from the slope of the line during an interval when velocity was increasing steadily.
4. Increase each mass by 10 g and repeat the observations. Continue increasing mass and finding acceleration until you have five total mass-acceleration data pairs.
5. Graph acceleration versus total mass. Draw a best-fit line through your data points.

Constant Total Mass

6. Set up a data table to record m_1, m_2, mass difference (Δm), Δd and Δt (if you use traditional equipment), and a.

7. Make $m_1 = 150$ g and $m_2 = 160$ g. Make observations to find the downward acceleration, using the same method as in step 3.

8. Transfer one 10 g mass from m_1 to m_2. The mass difference will now be 30 g, but the total mass will not have changed. Repeat your measurements.

9. Repeat step 8 until you have data for five mass difference-acceleration pairs.

10. Graph acceleration versus mass difference. Draw a best-fit line or curve through your data points.

Analyze and Conclude

1. Based on your graphs for step 5, what type of relationship exists between total mass and acceleration in an Atwood machine? Use appropriate curve-straightening techniques to support your answer (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write the relationship symbolically.

2. Based on your graphs for step 10, what type of relationship exists between mass difference and acceleration in an Atwood machine? Write the relationship symbolically.

3. How well do your results support your prediction?

4. String that is equal in length to the string connecting the masses over the pulley is sometimes tied to the bottoms of the two masses, where it hangs suspended between them. Explain why this would reduce experimental errors. Hint: Consider the mass of the string as the apparatus moves and how that affects m_1 and m_2.

5. Mathematical analysis shows that the acceleration of an ideal (frictionless) Atwood machine is given by $a = g\frac{m_1 - m_2}{m_1 + m_2}$. Use this relationship and your experimental results to find an experimental result for g.

6. Calculate experimental error in your value of g. Suggest the most likely causes of experimental error in your apparatus and procedure.

Apply and Extend

7. Start with Newton's second law in the form $\vec{a} = \frac{\vec{F}}{m}$ and derive the equation for a in question 5 above. Hint: Write $\vec{F}$ and m in terms of the forces and masses in the Atwood machine.

8. Using the formula $a = g\frac{m_1 - m_2}{m_1 + m_2}$ for an Atwood machine, find the acceleration when $m_1 = 2m_2$.

9. Under what circumstances would the acceleration of the Atwood machine be zero?

10. What combination of masses would make the acceleration of an Atwood machine equal to $\frac{1}{2}g$?

WEB LINK

www.mcgrawhill.ca/links/physics12

For some interactive activities involving the Atwood machine, go to the above Internet site and click on **Web Links**.

Assigning Direction to the Motion of Connected Objects

When two objects are connected by a flexible cable or rope that runs over a pulley, such as the masses in an Atwood machine, they are moving in different directions. However, as you learned when working with trains of objects, connected objects move as a unit. For some calculations, you need to work with the forces acting on the combined objects and the acceleration of the combined objects. How can you treat the pair of objects as a unit when two objects are moving in different directions?

Since the connecting cable or rope changes only the direction of the forces acting on the objects and has no effect on the magnitude of the forces, you can assign the direction of the motion as being from one end of the cable or rope to the other. You can call one end "negative" and the other end "positive," as shown in Figure 1.13.

When you have assigned the directions to a pair of connected objects, you can apply Newton's laws to the objects as a unit or to each object independently. When you treat the objects as one unit, you must ignore the tension in the rope because it does not affect the movement of the combined objects. Notice that the force exerted by the rope on one object is equal in magnitude and opposite in direction to the force exerted on the other object. However, when you apply the laws of motion to one object at a time, you must include the tension in the rope, as shown in the following sample problem.

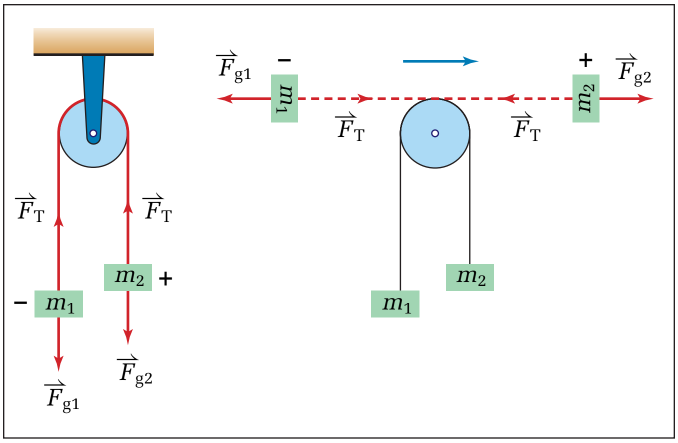


Figure 1.13 You can assign the bottom of the left-hand side of the machine to be negative and the bottom of the right-hand side to be positive. You can then imagine the connected objects as forming a straight line, with left as negative and right as positive. When you picture the objects as a linear train, make sure that you keep the force arrows in the same *relative* directions in relation to the individual objects.

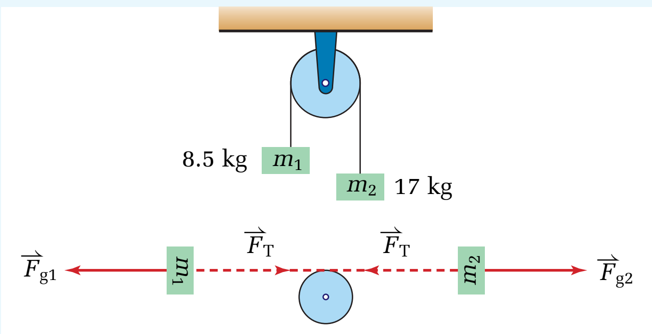
SAMPLE PROBLEM

Motion of Connected Objects

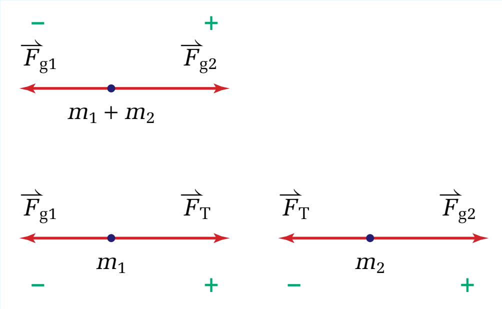
An Atwood machine is made of two objects connected by a rope that runs over a pulley. The object on the left (m_1) has a mass of 8.5 kg and the object on the right (m_2) has a mass of 17 kg.

(a) What is the acceleration of the masses?

(b) What is the tension in the rope?



Conceptualize the Problem



- To start framing the problem, draw free-body diagrams. Draw one diagram of the system moving as a unit and diagrams of each of the two individual objects.
- Let the *negative* direction point from the centre to the 8.5 kg mass and the *positive* direction point from the centre to the 17 kg mass.
- Both objects move with the same acceleration.
- The force of gravity acts on both objects.
- The tension is constant throughout the rope.
- The rope exerts a force of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the tension in the rope is one of the forces acting on the object.
- Newton's second law applies to the combination of the two objects and to each individual object.

Identify the Goal

(a) The acceleration, $\vec{a}$, of the two objects

(b) The tension, $|\vec{F}_T|$, in the rope

Identify the Variables

Known	Implied	Unknown
$m_1 = 8.5$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{g1}$ $\vec{F}_T$
$m_2 = 17$ kg		$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combination of masses to find the acceleration.

The mass of the combination is the sum of the individual masses.

$$\vec{F} = m\vec{a}$$

$$\vec{F}_{g1} + \vec{F}_{g2} = (m_1 + m_2)\vec{a}$$

$$-m_1 g + m_2 g = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$\vec{a} = \frac{(17\text{ kg} - 8.5\text{ kg})9.8\ \frac{\text{m}}{\text{s}^2}}{8.5\text{ kg} + 17\text{ kg}}$$

$$\vec{a} = 3.27\ \frac{\text{m}}{\text{s}^2}$$

$$\vec{a} \cong 3.3\ \frac{\text{m}}{\text{s}^2}\text{[to the right]}$$

(a) The acceleration of the combination of objects is 3.3 m/s^2 to the right.

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Apply Newton's second law to m_1 and solve for tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g1} + \vec{F}_T = m_1\vec{a}$$
$$-m_1 g + \vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = m_1 g + m_1\vec{a}$$
$$\vec{F}_T = (8.5\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (8.5\ \text{kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 111.18\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 1.1 \times 10^2\ \text{N}$$

(b) The tension in the rope is 1.1×10^2 N.

Validate the Solution

You can test your solution by applying Newton's second law to the second mass.

$$\vec{F}_{g2} + \vec{F}_T = m_2\vec{a}$$
$$m_2 g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - m_2 g$$
$$\vec{F}_T = (17\ \text{kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right) - (17\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = -111.18\ \text{N}$$
$$\vec{F}_T = -1.1 \times 10^2\ \text{N}$$

The magnitudes of the tensions calculated from the two masses independently agree. Also, notice that the application of Newton's second law correctly gave the direction of the force on the second mass.

PRACTICE PROBLEMS

19. An Atwood machine consists of masses of 3.8 kg and 4.2 kg. What is the acceleration of the masses? What is the tension in the rope?

20. The smaller mass on an Atwood machine is 5.2 kg. If the masses accelerate at 4.6 m/s^2, what is the mass of the second object? What is the tension in the rope?

21. The smaller mass on an Atwood machine is 45 kg. If the tension in the rope is 512 N, what is the mass of the second object? What is the acceleration of the objects?

22. A 3.0 kg counterweight is connected to a 4.5 kg window that freely slides vertically in its frame. How much force must you exert to start the window opening with an acceleration of 0.25 m/s^2?

23. Two gymnasts of identical 37 kg mass dangle from opposite sides of a rope that passes over a frictionless, weightless pulley. If one of the gymnasts starts to pull herself up the rope with an acceleration of 1.0 m/s^2, what happens to her? What happens to the other gymnast?

Objects Connected at Right Angles

In the lab, a falling weight is often used to provide a constant force to accelerate dynamics carts. Gravitational forces acting *downward* on the weight create tension in the connecting string. The pulley changes the direction of the forces, so the string exerts a *horizontal* force on the cart. Both masses experience the same acceleration because they are connected, but the cart and weight move at right angles to each other.

You can approach problems with connected objects such as the lab cart and weight in the same way that you solved problems involving the Atwood machine. Even if a block is sliding, with friction, over a surface, the mathematical treatment is much the same. Study Figure 1.14 and follow the directions below to learn how to treat connected objects that are moving both horizontally and vertically.

- Analyze the forces on each individual object, then label the diagram with the forces.
- Assign a direction to the motion.
- Draw the connecting string or rope as though it was a straight line. Be sure that the force vectors are in the same direction relative to each mass.
- Draw a free-body diagram of the combination and of each individual mass.
- Apply Newton's second law to each free-body diagram.

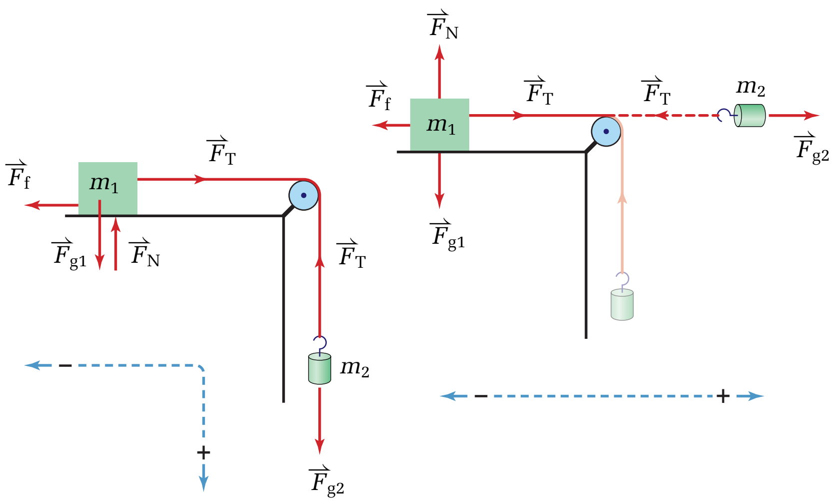


Figure 1.14 When you visualize the string "straightened," the force of gravity appears to pull down on mass 1, but to the side on mass 2. Although it might look strange, be assured that these directions are correct regarding the way in which the forces affect the motion of the objects.

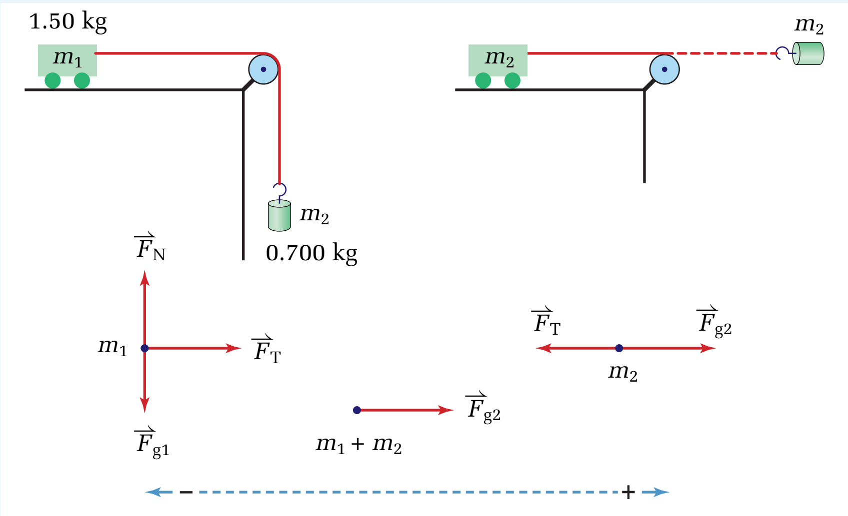
SAMPLE PROBLEM

Connected Objects

A 0.700 kg mass is connected to a 1.50 kg lab cart by a lightweight cable passing over a low-friction pulley. How fast does the cart accelerate and what is the tension in the cable? (Assume that the cart rolls without friction.)

Conceptualize the Problem

- Make a simplified diagram of the connected masses and assign forces.
- Visualize the cable in a straight configuration.
- Sketch free-body diagrams of the forces acting on each object and of the forces acting on the combined objects.



- The force causing the *acceleration* of both masses is the *force* of *gravity* acting on mass 2.
- Newton's second law applies to the combined masses and to each individual mass.
- Let left be the *negative* direction and right be the *positive* direction.

Identify the Goal

The acceleration of the cart, $\vec{a}$, and the magnitude of the tension force in the cable, F_T

Identify the Variables and Constants

Known	Implied	Unknown	
$m_1 = 1.50$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{a}$	$\vec{F}_{g1}$
$m_2 = 0.700$ kg		$\vec{F}_T$	$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combined masses and solve for acceleration.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g2} = (m_1 + m_2)\vec{a}$$
$$m_2 g = (m_1 + m_2)\vec{a}$$
$$\vec{a} = \frac{m_2 g}{m_1 + m_2}$$

Substitute values and solve.

$$\vec{a} = \frac{(0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{0.700\ \text{kg} + 1.5\ \text{kg}}$$
$$\vec{a} = 3.121\ 36\ \frac{\text{m}}{\text{s}^2}$$
$$\vec{a} \cong 3.1\ \frac{\text{m}}{\text{s}^2}$$

The cart accelerates at about 3.1 m/s^2. Since the sign is positive, it accelerates to the right.

Apply Newton's second law to mass 1 to find the tension in the rope.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = (1.5\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 4.682\ 04\ \text{N}$$
$$\vec{F}_T \cong 4.7\ \text{N}$$

The tension in the cable is about 4.7 N.

Validate the Solution

The acceleration of the combined masses is less than 9.81 m/s^2, which is reasonable since only part of the mass is subject to unbalanced gravitational forces. Also, the tension calculated at m_2 is also about 4.7 N.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - \vec{F}_g$$
$$\vec{F}_T = (0.700\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right) - (0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T \cong -4.7\ \text{N}$$

PRACTICE PROBLEMS

24. A Fletcher's trolley apparatus consists of a 1.90 kg cart on a level track attached to a light string passing over a pulley and holding a 0.500 kg mass suspended in the air. Neglecting friction, calculate

(a) the tension in the string when the suspended mass is released

(b) the acceleration of the trolley

25. A 40.0 g glider on an air track is connected to a suspended 25.0 g mass by a string passing over a frictionless pulley. When the mass is released, how long will it take the glider to travel the 0.85 m to the other end of the track? (Assume the mass does not hit the floor, so there is constant acceleration during the experiment.)

Free Fall

Figure 1.15 When you are on a free-fall amusement park ride, you feel weightless.

Have you ever dared to take an amusement park ride that lets you fall with almost no support for a short time? A roller coaster as it drops from a high point in its track can bring you close to the same feeling of **free fall**, a condition in which gravity is the only force acting on you. To investigate free fall quantitatively, imagine, once again, that you are standing on a scale in an elevator. If the cable was to break, there were no safety devices, and friction was negligible, what would be your apparent weight?

If gravity is the only force acting on the elevator, it will accelerate downward at the acceleration due to gravity, or g. Substitute this value into Newton's second law and solve for your apparent weight.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Let "up" be positive and "down" be negative. The total force acting on you is the downward force of gravity and the upward normal force of the scale. Your acceleration is g downward. $F_N + F_g = -mg$
- The force of gravity is $-mg$. $F_N - mg = -mg$
- Solve for the normal force. $F_N = mg - mg$
  $F_N = 0$

The reading on the scale is zero. Your apparent weight is zero. This condition is often called "weightlessness." Your mass has not changed, but you feel weightless because nothing is pushing up on you, preventing you from accelerating at the acceleration due to gravity.

Conceptual Problem

- How would a person on a scale in a freely falling elevator analyze the forces that were acting? Make a free-body analysis similar to the one in the sample problem (Apparent Weight) on page 28, using the elevator as your frame of reference. Consider these points.
  - **(a)** To an observer in the elevator, the person on the scale would not appear to be moving.
  - **(b)** The reading on the scale (the normal force) would be zero.

Close to Earth's surface, weightlessness is rarely experienced, due to the resistance of the atmosphere. As an object collides with molecules of the gases and particles in the air, the collisions act as a force opposing the force of gravity. **Air resistance** or air friction is quite different from the surface friction that you have studied. When an object moves through a fluid such as air, the force of friction increases as the velocity of the object increases.

A falling object eventually reaches a velocity at which the force of friction is equal to the force of gravity. At that point, the net force acting on the object is zero and it no longer accelerates but maintains a constant velocity called **terminal velocity**. The shape and orientation of an object affects its terminal velocity. For example, skydivers control their velocity by their position, as illustrated in Figure 1.16. Table 1.2 lists the approximate terminal velocities for some common objects.

Figure 1.16 Gravity is not the only force affecting these skydivers, who have become experts at manipulating air friction and controlling their descent.

Table 1.2 Approximate Terminal Velocities

Object	Terminal velocity (m/s downward)
large feather	0.4
fluffy snowflake	1
parachutist	7
penny	9
skydiver (spread-eagled)	58

PHYSICS FILE

In 1942, Soviet air force pilot I. M. Chisov was forced to parachute from a height of almost 6700 m. To escape being shot by enemy fighters, Chisov started to free fall, but soon lost consciousness and never opened his parachute. Air resistance slowed his descent, so he probably hit the ground at about 193 km/h, plowing through a metre of snow as he skidded down the side of a steep ravine. Amazingly, Chisov survived with relatively minor injuries and returned to work in less than four months.

TECHNOLOGY LINK

Air resistance is of great concern to vehicle designers, who can increase fuel efficiency by using body shapes that reduce the amount of air friction or drag that is slowing the vehicle. Athletes such as racing cyclists and speed skaters use body position and specially designed clothing to minimize drag and gain a competitive advantage. Advanced computer hardware and modelling software are making computerized simulations of air resistance a practical alternative to traditional experimental studies using scale models in wind tunnels.